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THE GENERAL THEORY OF NON-RECIPROCAL DIRECTIONAL COUPLERS AND APPLICATIONS IN TWR CIRCUITS

by

Henry Berger

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Rome Air Development Center
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FOREWORD

The author wishes to express his gratitude to Professor J. W. E. Griemsmann for his valuable discussions and criticisms of this work. He would also like to thank Professor M. Sucher for his critical comments. A large number of the ideas in this report were originally submitted in a term paper to Professor Sucher for his course on microwave components.

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* is presented.

ABSTRACT

~~This paper is concerned with~~ an analysis of non-reciprocal directional couplers and their application in traveling-wave resonant (TWR) circuits.* ~~The following is accomplished:~~

~~1.~~ The general characteristic of this type of coupler is determined from an examination of its scattering matrix. One result of interest is that it is not necessary that there be a $\pi/2$ phase difference between the output waves of the coupler resulting from a single input wave. This phase difference is necessary for symmetrical couplers.

~~2.~~ One type of non-reciprocal directional coupler is examined, and the various elements of the scattering matrix are related to its physical parameters.

~~3.~~ The non-reciprocally-coupled TWR circuit is analyzed for steady state and transient behavior, with and without the presence of reflections in the closed loop of the circuit. It is demonstrated that a variable non-reciprocal coupler can serve three functions simultaneously:

(1) isolate the power source from amplified ring reflections and hence permit stable tuning of the circuit;

(2) isolate the ring from the waves reflected from the dummy load; and

(3) act as a variable coupler which is needed to tune the circuit.

~~4.~~ From the manner in which the non-reciprocal character is permitted to occur, it is conjectured that it is not possible to devise a TWR circuit which makes use of any arrangement of 4-port directional couplers of any type which can store in the forward-ring wave all the energy supplied by the source, even when the dissipation in the ring loop can be neglected.

~~5.~~ The well-known theorem, which states that, if a lossless 4-port is matched, it must be a directional coupler, is found to be incorrect because it implicitly assumes reciprocity in its derivation. Such 4-ports which are non-reciprocal need not be directional couplers.

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Introduction

When it is desirable to divert a portion of the energy "flowing" in a given direction in a transmission line into a second transmission line in another given direction, a directional coupler is used. Theoretically such devices can be designed such that:

1. no reflections are set up in either transmission line due to the presence of the coupler;
2. the coupler is lossless;
3. energy is not diverted in a direction opposite to the desired one; and
4. the device is a 4-port.

In practical devices, the preceding characteristics can be almost completely realized and will be taken as our definition of a directional coupler.

If b is the column matrix of the reflected waves measured at the reference planes of the coupler, and a is the column matrix of the incident waves measured at the same reference planes, then

$$b = S.a \tag{1}$$

where S is the scattering matrix. For a symmetrical directional coupler S is well known and is given by

$$S = \begin{bmatrix} 0 & \sqrt{1-c^2} & 0 & jc \\ \sqrt{1-c^2} & 0 & jc & 0 \\ 0 & jc & 0 & \sqrt{1-c^2} \\ jc & 0 & \sqrt{1-c^2} & 0 \end{bmatrix} \tag{2}$$

The parameter c is called the voltage coupling coefficient and the reference planes are numbered as shown in Fig. 1.

If a unit amplitude wave impinges on port 1 of a directional coupler, a wave of amplitude and phase (relative to the phase of the wave emerging from port 2) jc will result at reference plane 4. If the directional coupler is reciprocal, then a unit amplitude wave incident at port 4 will similarly result in a wave jc at reference plane 1. If the directional coupler is non-reciprocal, then a wave with amplitude and phase other than jc will result at reference plane 1.

This paper is concerned with the analysis of non-reciprocal directional couplers and their application in traveling wave resonant (TWR) circuits. Such couplers may be viewed as imperfect 4-port circulators. In a perfect 4-port circulator, the energy injected in the i^{th} port will only emerge (with small insertion loss in a practical device) from the j^{th} port. When energy is injected in the j^{th} port, it will only emerge from the k^{th} port ($k \neq i \neq j$).

In a non-reciprocal directional coupler, a wave incident on one port can produce waves which exist simultaneously from two other ports of the device. The non-reciprocal character of the device is related to the fact that the coupling between two ports depends on into which of the two ports energy is injected.

General Properties

We shall first explore the general properties of a non-reciprocal directional coupler from an examination of the scattering matrix for such a device. With reference to Fig. 1, the properties of infinite directivity and zero reflections restrict the form of the scattering matrix to:

$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix} \qquad (3)$$

The condition of losslessness expressed in matrix form is the well-known relation[1]:

$$\tilde{S}\, S^{*} = 1 \qquad (4)$$

where $\tilde{S}$ equals the transpose of S, and S^{*} is the complex conjugate of S, and 1 is the unity matrix.

If $S_{ik} = \underline{S}_{ik} \exp(j\theta_{ik})$, where $j = \sqrt{-1}$ and $\underline{S}_{ik} = |S_{ik}|$, then the application of eq. (4) to eq. (3) yields the following phase information:

$$(\theta_{32} - \theta_{12}) + (\theta_{14} - \theta_{34}) = \pm \pi \qquad (5)$$

and

$$(\theta_{41} - \theta_{21}) + (\theta_{23} - \theta_{43}) = \pm \pi \qquad (6)$$

It should be understood that the summations in equations (5) and (6) are ambiguous by a factor $2n\pi$. ($n = \pm 1, \pm 2, \pm 3, \ldots$)

Equations (5) and (6) reveal that the phase difference between the output waves, caused by any single input wave, does not necessarily equal $\pi/2$ radians separately, as is true for reciprocal directional couplers. Rather, the sum of various combinations of such phase differences equal π radians. If $(\theta_{41} - \theta_{21}) = (\theta_{23} - \theta_{43})$, and $(\theta_{32} - \theta_{12}) = (\theta_{14} - \theta_{34})$, then the output phase differences are $\pi/2$ radians separately, whether or not the coupler is reciprocal.

Another interesting result is obtained by subtracting eq. (6) from eq. (5)

$$(\theta_{41} - \theta_{14}) + (\theta_{23} - \theta_{32}) + (\theta_{12} - \theta_{21}) + (\theta_{34} - \theta_{43}) = 0 \qquad (7)$$

Similar to the previous case, the bracketed terms are not necessarily separately zero, as is true for reciprocal directional couplers.

If we choose for convenience $\underline{S}_{41}$ and $\underline{S}_{14}$ as our independent parameters, the application of eq. (4) to eq. (3) yields relations which allow us to express all other amplitudes (of the matrix elements) in terms of these parameters. The results are:

$$\begin{array}{ll} \underline{S}_{21} = \sqrt{1 - \underline{S}_{41}^{2}} & \underline{S}_{32} = \underline{S}_{14} \\ \underline{S}_{23} = \underline{S}_{41} & \underline{S}_{34} = \sqrt{1 - \underline{S}_{14}^{2}} \\ \underline{S}_{12} = \sqrt{1 - \underline{S}_{14}^{2}} & \underline{S}_{41} = \underline{S}_{41} \\ \underline{S}_{14} = \underline{S}_{14} & \underline{S}_{43} = \sqrt{1 - \underline{S}_{41}^{2}} \end{array} \qquad (8)$$

Thus the conditions of infinite directivity and zero reflections determine eight of the original sixteen complex elements of S. The added condition of losslessness reduces the number of independent amplitudes to two, for the remaining eight elements and independent phase angles to six of the eight remaining phase angles.

The preceding results can be derived as follows. If the operations implicity in $\underline{S}\ S^{*} = 1$ are performed, as shown below:

$$\begin{vmatrix} 0 & S_{21} & 0 & S_{41} \\ S_{12} & 0 & S_{32} & 0 \\ 0 & S_{23} & 0 & S_{43} \\ S_{14} & 0 & S_{34} & 0 \end{vmatrix} \cdot \begin{vmatrix} 0 & S_{12}^{*} & 0 & S_{14}^{*} \\ S_{21}^{*} & 0 & S_{23}^{*} & 0 \\ 0 & S_{32}^{*} & 0 & S_{34}^{*} \\ S_{41}^{*} & 0 & S_{43}^{*} & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} (5) \qquad (9)$$

then one obtains from an examination of the diagonal elements

$$|S_{21}|^2 + |S_{41}|^2 = 1 \tag{10}$$
$$|S_{12}|^2 + |S_{32}|^2 = 1 \tag{11}$$
$$|S_{23}|^2 + |S_{43}|^2 = 1 \tag{12}$$
$$|S_{14}|^2 + |S_{34}|^2 = 1 \tag{13}$$

and, from the off-diagonal elements,

$$S_{21}\ S_{23}^* + S_{41}\ S_{43}^* = 0 \tag{14}$$
$$S_{12}\ S_{14}^* + S_{32}\ S_{34}^* = 0 \tag{15}$$

as well as the redundant equations

$$S_{23}\ S_{21}^* + S_{43}\ S_{41}^* = 0$$
$$S_{14}\ S_{12}^* + S_{34}\ S_{32}^* = 0$$

Equation (14) may then be written

$$\underline{S}_{21}\ \underline{S}_{23}\ e^{j(\theta_{21} - \theta_{23})} + \underline{S}_{41}\ \underline{S}_{43}\ e^{j(\theta_{41} - \theta_{43})} = 0 \tag{16}$$

or

$$\underline{S}_{21}\ \underline{S}_{23}/\underline{S}_{41}\ \underline{S}_{43} = e^{j\left[(\theta_{41} - \theta_{21}) + (\theta_{23} - \theta_{43}) \pm \pi\right]} \tag{17}$$

Since the left side of eq. (17) is a real number, then the total phase angle in the exponential on the right side of eq. (17) must be the result given in eq. (5), i.e.:

$$(\theta_{41} - \theta_{21}) + (\theta_{23} - \theta_{43}) = \pm\ \pi\ \pm\ 2n\pi\ (n = \pm\ 1, \pm\ 2, \ldots) \tag{18}$$

In a similar manner, eq. (15) may be written

$$\underline{S}_{12}\ \underline{S}_{14}/\underline{S}_{23}\ \underline{S}_{34} = e^{j\left[(\theta_{32} - \theta_{12}) + (\theta_{14} - \theta_{34}) \pm \pi\right]} \tag{19}$$

and by a similar argument we obtain the result given in eq. (6).

To obtain the relations between the various amplitudes S_{ik}, we first substract eq. (12) from eq. (10) to get

$$(\underline{S}_{21})^2 - (\underline{S}_{23})^2 = (\underline{S}_{43})^2 - (\underline{S}_{41})^2 \tag{20}$$

However, we have from eq. (17)

$$\underline{S}_{21}\ \underline{S}_{23} = \underline{S}_{41}\ \underline{S}_{43} \tag{21}$$

Hence eq. (20) may be combined with eq. (21) to yield

$$\underline{S}_{21}^{2} - \underline{S}_{23}^{2} = \underline{S}_{43}^{2} - \underline{S}_{21}^{2}\ \underline{S}_{23}^{2}/\underline{S}_{43}^{2} \tag{22}$$

or

$$\underline{S}_{21}^{2}(1 + \underline{S}_{23}^{2}/\underline{S}_{43}^{2}) = \underline{S}_{43}^{2} + \underline{S}_{23}^{2} = \underline{S}_{43}^{2}(1 + \underline{S}_{23}^{2}/\underline{S}_{43}^{2}) \tag{23}$$

Thus

$$\underline{S}_{21} = \underline{S}_{43} \tag{24}$$

By a similar argument we can also get

$$\underline{S}_{12} = \underline{S}_{34} \tag{25}$$

Combining eqs. (24) and (21) we get

$$\underline{S}_{43}\ \underline{S}_{23} = \underline{S}_{41}\ \underline{S}_{43} \tag{26}$$

or

$$\underline{S}_{23} = \underline{S}_{41} \tag{27}$$

In a similar manner we can get

$$\underline{S}_{14} = \underline{S}_{32} \tag{28}$$

Furthermore we get, directly from eqs. (10) and (13)

$$\underline{S}_{21} = \sqrt{1 - \underline{S}_{41}^{2}} \tag{29}$$

$$\underline{S}_{34} = \sqrt{1 - \underline{S}_{14}^{2}} \tag{30}$$

Finally, combining eqs. (11) and (28); and (12) and (27), we get:

$$\underline{S}_{12} = \sqrt{1 - \underline{S}_{14}^{2}} \tag{31}$$

$$\underline{S}_{43} = \sqrt{1 - \underline{S}_{41}^{2}} \tag{32}$$

In our definition of a directional coupler, stated in the introduction, condition (3) prescribes complete isolation between two pairs of adjacent ports. For the scattering matrix in eq. (3) ports one and three were assumed completely isolated from one another, as well as ports two and four.

If, instead, we had considered matched, lossless, four ports, the preceding derivations would have required ten simultaneous equations involving amplitudes of the twelve non-zero scattering matrix elements. These equations have non-trivial solutions. Hence the well known theorem which states that a lossless, matched four port is necessarily a directional coupler (i.e., has two pairs of adjacent, isolated ports) is incorrect. The error arises because the derivation implicitly assumes reciprocity.

It is of interest to note the manner in which the non-reciprocal character is permitted to display itself. For example, the coupling coefficients C_{41} and C_{23} must be equal, and similarly C_{14} and C_{32} must be equal. The lack of reciprocity is only permitted to occur by having $C_{41} \neq C_{14}$ and/or $C_{32} \neq C_{23}$. See eqs. (27) and (28).

With the above restrictions in mind consider the problem of constructing some directional coupler such that when it is used to provide the coupling action for a traveling wave resonant loop,(see Fig. 3) the loop will receive and store all the energy delivered by the power source even during the transient build-up. A detailed analysis of such a loop which uses a non-reciprocal directional coupler is presented in a later section. However, it is readily seen from the general discussion given in this section that we would require $C_{23} = 0$ and $C_{41} = 1$, and we have shown this is impossible.

One can further conjecture that it is impossible to devise any arrangement of couplers of any type such that the energy normally lost during the transient build-up of the loop would instead be received and stored within the loop in the foward ring wave.

An Example of a Non-Reciprocal Directional Coupler

We shall now analyze a specific non-reciprocal directional coupler with relatively high power handling capacity. The application of this coupler in high power TWR circuits will be explored in later sections.

A well known circuit shall be called here the cascaded directional coupler type (CDCT) and is illustrated in Fig. 2. The difference of the phase lengths of the two sections of waveguide which connected together the directional couplers must be different for waves traveling in one direction compared to waves traveling in the opposite direction. One way this can be achieved is by loading the waveguides with appropriately positioned bars of ferrite material.

We shall assume for simplicity that the voltage coupling coefficients of the two couplers are equal. The loss and phase shift between the couplers are described by:

$$a_{3'} = b_{4'} A_{3'4'} e^{j\theta_{3'4'}} \qquad a_{4'} = b_{3'} A_{4'3'} e^{j\theta_{4'3'}}$$
$$a_{1'} = b_{2'} A_{1'2'} e^{j\theta_{1'2'}} \qquad a_{2'} = b_{1'} A_{2'1'} e^{j\theta_{2'1'}} \tag{33}$$

where $A_{i'k'}$ and $\theta_{i'k'}$ are the loss factor and phase lengths respectively of the length of line between ports i' and k'. $(A_{i'k'} \leq 1)$

If eq. (33) is used in conjunction with eq. (21), the overall scattering matrix S_{CDCT} for a CDCT can be readily derived. Its non-zero elements are given below:

$$S_{21} = -c^2 A_{3'4'} e^{j\theta_{3'4'}} + (1-c^2) A_{1'2'} e^{j\theta_{1'2'}}$$
$$S_{23} = jc\sqrt{1-c^2}\left(A_{3'4'} e^{j\theta_{3'4'}} + A_{1'2'} e^{j\theta_{1'2'}}\right)$$
$$S_{12} = -c^2 A_{4'3'} e^{j\theta_{4'3'}} + (1-c^2) A_{2'1'} e^{j\theta_{2'1'}}$$
$$S_{14} = jc\sqrt{1-c^2}\left(A_{2'1'} e^{j\theta_{2'1'}} + A_{4'3'} e^{j\theta_{4'3'}}\right) \tag{34}$$
$$S_{32} = jc\sqrt{1-c^2}\left(A_{4'3'} e^{j\theta_{4'3'}} + A_{2'1'} e^{j\theta_{2'1'}}\right)$$
$$S_{34} = -c^2 A_{2'1'} e^{j\theta_{2'1'}} + (1-c^2) A_{4'3'} e^{j\theta_{4'3'}}$$
$$S_{41} = jc\sqrt{1-c^2}\left(A_{1'2'} e^{j\theta_{1'2'}} + A_{3'4'} e^{j\theta_{3'4'}}\right)$$
$$S_{43} = -c^2 A_{1'2'} e^{j\theta_{1'2'}} + (1-c^2) A_{3'4'} e^{j\theta_{3'4'}}$$

Eq. (34) includes the effect of losses. If we now assume lossless phase shifting elements, and 3 db hybrids (i.e., $c = 1/\sqrt{2}$), eq. (34) becomes:

$$
\begin{aligned}
S_{21} &= 1/2\left(e^{j\theta_{1'2'}} - e^{j\theta_{3'4'}}\right) = e^{j\theta_{1'2'}}\left(1 - e^{jD}\right)/2 \\
S_{12} &= 1/2\left(e^{j\theta_{2'1'}} - e^{j\theta_{4'3'}}\right) = e^{j\theta_{2'1'}}\left(1 - e^{jB}\right)/2 \\
S_{41} &= je^{j\theta_{1'2'}}\left(e^{jD} + 1\right)/2 \\
S_{14} &= je^{j\theta_{2'1'}}\left(e^{jB} + 1\right)/2 \\
S_{23} &= je^{j\theta_{1'2'}}\left(1 + e^{jD}\right)/2 \\
S_{32} &= je^{j\theta_{2'1'}}\left(1 + e^{jB}\right)/2 \\
S_{43} &= e^{j\theta_{1'2'}}\left(e^{jD} - 1\right)/2 \\
S_{34} &= e^{j\theta_{2'1'}}\left(e^{jB} - 1\right)/2
\end{aligned}
\qquad \Omega \qquad (35)
$$

where $D = \theta_{3'4'} - \theta_{1'2'}$, and $B = \theta_{4'3'} - \theta_{2'1'}$.

If a unit amplitude wave is incident on port 1, then a wave of amplitude,

$$C_{41} = |S_{41}| = |\left(1 + e^{jD}\right)/2| = \cos D/2$$

will emerge from port 4. But if a unit amplitude wave is incident on port 4, then a wave of amplitude

$$|C_{14} = |S_{14}| = |\left(1 + e^{jB}\right)/2| = \cos B/2$$

will emerge from port 1.

Thus the "forward" and "backward" voltage coupling coefficients apparently can be chosen independently. For example, if $D = 0$ and $B = \pi$, then $C_{41} = 1$, $C_{14} = 0$. For this situation the device is acting as a perfect circulator. However, one can design the device so that $D = 0$ and $B = \pi/2$, then $C_{41} = 1$, $C_{14} = 1/\sqrt{2}$. Thus one can, in general, consider a non-reciprocal directional coupler to be an imperfect circulator.

A circuit somewhat similar to the preceding one, and commonly used in high power duplexer applications, is one in which one directional coupler is replaced by a magic tee (generally of the folded type for ease of packaging). For this circuit it can be shown that, neglecting losses, the foward and reverse coupling coefficients for ports 1 and 4 would now be

$$C_{41} = \left(\frac{1 + \sin D}{2}\right)^{1/2}$$

and

$$C_{14} = \left(\frac{1 + \sin B}{2}\right)^{1/2}$$

In this duplexing circuit the ferrite bars are positioned so that the ferrite-loaded interconnecting waveguide are mirror images of one another with respect to a plane parallel to the waveguides axes and equally spaced between them. This geometry leads to the non-reciprocal phase condition $D = -B$ assuming the ferrite bars are identical and subject to identical magnetic fields. The coupling coefficients are then

$$C_{41} = \left(\frac{1 + \sin D}{2}\right)^{1/2}$$

and

$$C_{14} = \left(\frac{1 - \sin D}{2}\right)^{1/2}$$

Thus $(C_{14})^2 + (C_{41})^2 = 1$. When D is chosen as $\pi/2$ radians, by proper adjustment of the biasing magnetic field, then $C_{41} = 1$, and $C_{14} = 0$, and the device functions as a circulator. (Notice that for the CDCT the condition $D = -B$ would not introduce non-reciprocal coupling action because the coupling coefficients are even (cosine) functions of the phase difference.)

The preceding, however, would have little practical use as a non-reciprocal directional coupler because by making B dependent on D, we lose the option of choosing C_{41} independently of the choice of C_{14}. To avoid this difficulty one must avoid the mirror symmetry described above and attempt to maintain two independent controls, one on the forward phase differential and one on the backward phase differential.

Applications to High Power TWR Circuits

A. Steady State Analysis for a Reflectionless Ring

The non-reciprocal property of a directional coupler would be of value whenever it is desired to couple heavily from one transmission line to a second and yet not permit reflections in the second (coupled) line to couple back into the primary line. One such situation is in the use of a TWR high-power, breakdown, microwave test circuit (see Fig. 3).

TWR circuits have been extensively analyzed.[2,3,4] The basic phenomena of traveling wave resonance may be pictured as follows. A wave traveling down a transmission line is partially coupled into the closed path when it encounters a directional coupler (see Fig. 3). The coupled wave travels around the closed path being attenuated and phase-shifted as it does. When the coupled wave has circled the closed path and reaches the coupler from which it entered, some fraction of it couples back into the original transmission line.

The fraction of the wave remaining in the closed path adds vectorially to the wave newly coupled in through the directional coupler. If these waves are in phase with each other, constructive addition occurs. If the wave are π radians out of phase, destructive addition occurs. This process continues until the circuit reaches equilbrium. By appropriate choice of coupling coefficient and electrical length of the closed path, the amplitude of resultant wave in the closed path may be varied over a considerable latitude, from many times the input wave from the external power source,down to a fraction of the input wave.

TWR circuits which have sufficiently low attenuation so that they may be tuned to produce average powers flowing within the closed path of the circuit which are many times greater than the average power supplied by the source, are utilized for high-power microwave breakdown testing. Such devices are used when the available power sources cannot produce sufficient power to perform adequate breakdown tests when this power is directly applied to a component. Reflections caused by discontinuities in the closed loop of the TWR circuit may be amplified manifold. In fact, it is possible under certain conditions that small discontinuities in the loop may have almost the same effect as a perfect short placed in the primary line. In practice this reflection amplification more often than not makes the tuning of such a circuit extremely difficult.

In the above situation, a non-reciprocal directional coupler would serve three purposes. First, it would function as a variable coupler, replacing the usual combination of three 3 db hybrids plus a pair of plungers that is normally used for this purpose. (The control of the coupling coefficient would now come from the control of the magnetic bias on the ferrites producing the phase shift.) Secondly, it would function as an isolator, protecting the power source from the undesired amplified reflections and hence, permitting simple stable tuning of the test circuit. Third, it can, at the same time as performing the first two functions, prevent reflections from the dummy load from adding power to the backward waves in the loop. Notice the non-reciprocal directional coupler is not being used as a circulator. In a circulator, the voltage coupling coefficient would essentially be unity, and this would make the tuning of the loop impossible. A steady state analysis of a reflectionless loop (no discontinuities) non-reciprocally coupled to a transmission line may be readily made.

We shall assume that all lines are terminated in matched loads. For convenience, relative phase lengths will be used in the analysis. This can be done by shifting reference plane 4 the electrical distance θ_{43}, and reference plane 2 the electrical distance θ_{21}. Analystically, these transformations are given by[5]

$$S' = P.S.P. \tag{36}$$

where P is given by

$$P = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{+j\theta_{21}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{+j\theta_{43}} \end{vmatrix} \tag{37}$$

Then b_4 can be written

$$b_4 = \underline{S}_{41}\, e^{j(\theta_{41} - \theta_{43})}\, a_1 + \sqrt{1 - \underline{S}_{41}^{\,2}}\; a_3 \tag{38}$$

If the attenuation factor for the closed loop of the TWR circuit is A, and the phase length is D, then

$$a_3 = A\, e^{jD} \tag{39}$$

Eqs. (38) and (39) can be combined to yield

$$b_4 = \frac{\underline{S}_{41}\, e^{j(\theta_{41} - \theta_{43})}\, a_1}{1 - \sqrt{1 - \underline{S}_{41}^{\,2}}\; A\, e^{jD}} \tag{40}$$

Eq. (40) is precisely the same form as the corresponding expression for a TWR circuit with a reciprocal directional coupler. Thus the same results must follow. For example, the power gain M_f^2, from eq. (40) is

$$M_f^2 = \left| \frac{b_4}{a_1} \right|^2 = \frac{\underline{S}_{41}^{\,2}}{1 - 2A\sqrt{1 - \underline{S}_{41}^{\,2}}\cos D + A^2 (1 - \underline{S}_{41}^{\,2})} \tag{41}$$

By straightforward differentiation, it can be shown that M^2 is maximum when $D = 2\pi n$ ($n = 1, 2, \ldots$), and $\underline{S}_{41}^{\,2} = 1 - A^2$. Then

$$M^2_{max} = \frac{1}{\underline{S}_{41}^{\,2}} = \frac{1}{1 - A^2} \tag{42}$$

As in the TWR circuit with reciprocal couplers, it is always possible to have the average power flowing in the closed path of the TWR circuit greater than the average power supplied by the generator (since $0 \leq \underline{S}_{41}^{2} = 1 - A^2 < 1$ for $0 \leq A < 1$). This, of course, is due to energy storage (as in a standing wave cavity) and not from any real amplification.

The principle of conservation of energy can be applied to the non-reciprocally-coupled TWR circuit and eqs. (5) and (6) concerning the necessary phase relationships for non-reciprocal directional couplers can be rederived in an instructive manner. For the TWR circuit the principle may be stated as follows: P_S (power supplied by the source) = P_R (power lost through dissipation in the closed loop) + P_L (power lost to the dummy load). We will continue to assume the absence of reflections.

Now $P_R = |b_4|^2 - |a_3|^2 = |b_4|^2 - A^2 |b_4|^2 = |b_4|^2 (1 - A^2) = M^2 a_1^{\,2} (1 - A^2)$.

(These are normalized voltages.) Hence:

$$P_S = |a_1|^2 = M^2 |a_1|^2 (1 - A^2) + P_L$$

or

$$1 = M^2 (1 - A^2) + P_L/P_S \tag{43}$$

Now if $M^2 = M^2\,(\max) = 1/(1 - A^2)$ (where $D = 2\pi n$, $\underline{S}_{41} = 1 - A^2$) then eq. (43) yields

$$P_L = 0 \text{ (when } D = 2\pi n,\ \underline{S}_{41}^{\,2} = 1 - A^2) \tag{44}$$

Now $P_L = |b_2|^2$, and

$$\begin{aligned} b_2 &= \sqrt{1 - \underline{S}_{41}^{\,2}}\ a_1 + \underline{S}_{14}\ e^{j(\theta_{23} - \theta_{21})}\ a_3 \\ &= \sqrt{1 - \underline{S}_{41}^{\,2}}\ a_1 + \underline{S}_{14}\ e^{j(\theta_{23} - \theta_{21})}\ A\, e^{jD}\, b_4 \\ &= \sqrt{1 - \underline{S}_{41}^{\,2}}\ a_1 + \underline{S}_{14} \left[e^{j(\theta_{23} - \theta_{21})} \right] \frac{A\, e^{jD}\, \underline{S}_{41}\, a_1\, e^{j(\theta_{41} - \theta_{43})}}{(1 - \sqrt{1 - \underline{S}_{41}^{\,2}}\ A\, e^{jD})} \end{aligned} \tag{45}$$

Then, evaluating eq. (45) where $D = 2\pi n$, $\underline{S}_{41}^2 = 1 - A^2$, $M^2 = 1/\underline{S}_{41}^2$, $\underline{S}_{41} = \underline{S}_{14}$

$$b_2 = A\, a_1 \left[1 + e^{j(\theta_{41} - \theta_{21}) + j(\theta_{23} - \theta_{43})}\right] \tag{46}$$

one can obtain

$$P_L = A^2 P_S \left|1 + e^{j(\theta_{41} - \theta_{21}) + j(\theta_{23} - \theta_{43})}\right|^2 \tag{47}$$

Since P_L is zero at $D = 2\pi n$, $\underline{S}_{41}^2 = 1 - A^2$, eq. (47) implies

$$(\theta_{41} - \theta_{21}) + (\theta_{23} - \theta_{43}) = \pm \pi \tag{48}$$

This is identical with eq. (5). In a similar manner eq. (6) can be derived.

Waves impinging on the coupler from the opposite direction, i.e., on port 2, produce a buildup in the loop described by the gain relation

$$M_b^2 \equiv \left|\frac{b_3}{a_2}\right|^2 = \frac{\underline{S}_{14}^2}{1 - 2A\sqrt{1 - \underline{S}_{14}^2}\,\cos D + A^2 (1 - \underline{S}_{14}^2)} \tag{49}$$

Notice that we can make $\underline{S}_{14} = 0$ and hence prevent any building of backward waves due to reflections from the dummy load. Since the gain relation in eq. (41) involves only $\underline{S}_{41}$ this will not affect the forward wave buildup. Of course, choosing $\underline{S}_{14} = 0$ also isolates the source from the effects of reflections in the closed loop.

B. Steady State Analysis for a Ring with Reflections

It was pointed out in Part A of this section that one value of using a non-reciprocal coupler with a TWR circuit is that it can be used to protect the source from the large backward waves which can build up in a tuned TWR circuit. In this section we shall analyze the build-up of reflected waves in a TWR circuit which utilizes non-reciprocal directional coupling.

To simplify the analysis, we shall assume that only a single source of reflections exists in the closed loop of the TWR circuit, and that it is described by some scattering matrix:

$$\begin{bmatrix} S_{55} & S_{56} \\ S_{65} & S_{66} \end{bmatrix} \tag{50}$$

Reference planes 5 and 6 are shown in Fig. 5. We shall describe the electrical lengths between reference planes 4 and 6 and between 5 and 3 by θ_6 and θ_5 respectively. If, in these lengths, no attenuation occurs, θ_5 and θ_6 will be real. If attenuation does occur, the appropriate θ will be complex to take this into account.

The forward and back wave (denoted by a_i and b_i) within the closed ring can be related, by inspection of Fig. 5, as follows:

$$\begin{aligned} a_6 &= b_4\, e^{-j\theta_6} \qquad & a_5 &= b_3\, e^{-j\theta_3} \\ a_4 &= b_6\, e^{-j\theta_6} \qquad & a_3 &= b_5\, e^{-j\theta_5} \end{aligned} \tag{51}$$

Then, utilizing eqs. (50) and (51), we have

$$b_5 = S_{55}\, a_5 + S_{56}\, a_6 = S_{55}\, b_5\, e^{-j\theta_5} + S_{56}\, b_4\, e^{-j\theta_6} \tag{52}$$

$$b_6 = S_{65}\, a_5 + S_{66}\, a_6 = S_{65}\, b_3\, e^{-j\theta_5} + S_{66}\, b_4\, e^{-j\theta_3} \tag{53}$$

To derive the voltage gain M, within the ring, of the forward wave in the ring ($M_f \equiv |b_4/a_1|$), we proceed to relate b_4 to a_1. Now from the scattering matrix of the coupler

$$b_4 = \underline{S}_{41}\, e^{-j\theta_{41}}\, a_1 + \sqrt{1 - \underline{S}_{41}^{\,2}}\; e^{-j\theta_{43}}\, a_3 \tag{54}$$

while from eqs. (50) and (51)

$$a_3 = b_5\, e^{-j\theta_5} = S_{55}\, b_3\, e^{-j2\theta_5} + S_{56}\, b_4\, e^{-j(\theta_5 + \theta_6)} \tag{55}$$

Eqs. (54) and (55) may be combined to yield

$$b_4 = \underline{S}_{41}\, e^{-j\theta_{41}}\, a_1 + \sqrt{1 - \underline{S}_{41}^{\,2}}\; e^{-j\theta_{43}} \left\{ S_{55}\, b_3\, e^{-2j\theta_5} + S_{56}\, b_4\, e^{-j(\theta_5 + \theta_6)} \right\} \tag{56}$$

All that is now necessary is to eliminate b_3 from eq. (56). To do this we observe that from eqs. (51) and (53), the scattering matrix of the coupler, and the assumption that there are no reflection from the dummy load, one gets

$$b_3 = \sqrt{1 - \underline{S}_{14}^{\,2}}\; e^{-j\theta_{34}} \left\{ b_6\, e^{-j\theta_6} \right\} \tag{57}$$

and hence

$$b_3 = \sqrt{1 - \underline{S}_{14}^2}\, e^{-j(\theta_{34} + \theta_{43})} \left\{ S_{65}\, b_3\, e^{-j\theta_5} + S_{66}\, b_4\, e^{-j\theta_6} \right\} \tag{58}$$

Eq. (58) can be rewritten as

$$b_3 = b_4 \left\{ \frac{\sqrt{1 - \underline{S}_{14}^2}\, e^{-j(\theta_{34} + 2\theta_6)} S_{66}}{1 - \sqrt{1 - \underline{S}_{14}^2}\, e^{-j(\theta_{34} + \theta_6 + \theta_5)} S_{65}} \right\} \tag{59}$$

If Eq. (59) is now substituted into eq. (56), one obtains

$$b_4 = \underline{S}_{41}\, e^{-j\theta_{41}} a_1 + \sqrt{1 - \underline{S}_{41}^2}\, e^{-j\theta_{43}} \left\{ \frac{S_{55} S_{66} \sqrt{1 - \underline{S}_{14}^2}\, e^{-j(\theta_{34} + 2\theta_6 + 2\theta_5)}}{1 - \sqrt{1 - \underline{S}_{14}^2}\, S_{65}\, e^{-j(\theta_{34} + \theta_5 + \theta_6)}} + S_{56}\, e^{-j(\theta_5 + \theta_6)} \right\} \tag{60}$$

Eq. (60) may now be solved for the ratio of b_4 to a_1, and hence the forward gain, M_f of the TWR circuit is

$$M_f = \frac{b_4}{a_1} = \frac{\underline{S}_{41}\, e^{-j\theta_{41}} \left[1 - \sqrt{1 - \underline{S}_{14}^2}\, S_{65}\, e^{-j(\theta_{31} + \theta_5 + \theta_6)} \right]}{E} \tag{61}$$

where

$$\begin{aligned} E \equiv 1 &- (S_{65} S_{56} - S_{55} S_{66}) \sqrt{1 - \underline{S}_{41}^2} \sqrt{1 - \underline{S}_{14}^2}\, e^{-j(\theta_{34} + \theta_{43} + 2\theta_5 + 2\theta_6)} \\ &- S_{56} \sqrt{1 - \underline{S}_{41}^2}\, e^{-j(\theta_{43} + \theta_5 + \theta_6)} - S_{65} \sqrt{1 - \underline{S}_{14}^2}\, e^{-j(\theta_{34} + \theta_5 + \theta_5)} \end{aligned} \tag{62}$$

When the coupler is symmetrical ($\underline{S}_{14} = \underline{S}_{41}$, $\theta_{34} = \theta_{43}$), Eq. (61) reduces to the standard result for a TWR circuit. It is obvious from eq. (60) that the presence of reflections in the closed ring can affect the forward gain of the ring to a considerable extent. The reflection coefficient within the ring, ($R_r \equiv a_4/b_4$) and hence the VSWR within the ring can be readily determined. From eq. (59) and the coupler scattering matrix, we have

$$\begin{aligned} a_4 &= \frac{e^{+j\theta_{34}}}{\sqrt{1 - S_{14}^2}}\, b_3 \\ &= \frac{e^{+j\theta_{34}}}{\sqrt{1 - \underline{S}_{14}^2}} \frac{\sqrt{1 - \underline{S}_{14}^2}\, e^{-j(\theta_{34} + 2\theta_6)} S_{66}\, b_4}{\left(1 - \sqrt{1 - \underline{S}_{14}^2}\, e^{-j(\theta_{34} + \theta_5 + \theta_6)} S_{65}\right)} \end{aligned} \tag{63}$$

The internal ring reflection coefficient R_r is calculated from eq. (63) to be

$$R_r \cong \frac{a_4}{b_4} = \frac{S_{66}\, e^{-2j\theta_6}}{1 - \sqrt{1 - \underline{S}_{14}^{\,2}}\; e^{-j(\theta_{34} + \theta_5 + \theta_6)}\, S_{65}} \tag{64}$$

Now S_{66} equals the reflection coefficient of the device, which is the source of reflections in the ring, seen at reference plane 6. Hence, eq. (64) demonstrates that, if $S_{65} \cong 1$, the ring reflection coefficient R_r is directly proportional to the reflection coefficient of the reflecting device seen at port 6. Note, however, that R_r is not identically equal to S_{66} as might be intuitively expected. Depending on the phases in eq. (64) and the amplitude $\underline{S}_{14}$, R_r may be much greater or lesser than S_{66}.

For example, if $\theta_{34} + \theta_5 + \theta_6 \cong 2n\pi$, $S_{65} \cong 1$, and $\mathrm{Im}\,(\theta_6) \cong \mathrm{Im}(\theta_5) \cong 0$ then, using eq. (49) with A = 1, D = $2n\pi$, we get

$$R_r \cong \frac{S_{66}\, M_b}{\underline{S}_{14}} \tag{65}$$

Now for a reflectionless reciprocally-coupled TWR circuit $\underline{S}_{14}^{\,2} = \underline{S}_{41}^{\,2} = 1/M^2\,\mathrm{max}$, (note $\underline{S}_{14}^{\,2} = \underline{S}_{41}^{\,2} \neq 0$, for the steady state amplitude of the ring wave to be greater than 0) and hence if the ring is tuned for maximum gain with $S_{66} << 1$:

$$R_r \cong (M)^2_{max}\; S_{66} \tag{66}$$

Eq. (66) states that in a reciprocally coupled ring the reflection coefficient of a reflective device integrated into the closed loop of a TWR circuit is amplified by a factor equal to the power amplification gain in the ring. The situation is different for a non-reciprocally-coupled ring, in which case $\underline{S}_{14}^{\,2}$ may equal zero. Since $\underline{S}_{14}^{\,2} \neq \underline{S}_{41}^{\,2} = 1/(M_f)^2$ max, and $S_{14}^{\,2} \neq 1/(M_b)^2$ max, (since M_b^2 refers to the gain of a ring wave created by energy reflected from the dummy load) setting $S_{14}^{\,2}$ equal to zero is permissable. If $\underline{S}_{14}^{\,2}$ is zero, then

$$R_r = \frac{S_{66}\, e^{-2j\theta_6}}{1 - e^{-j(\theta_{34} + \theta_5 + \theta_6)}\, S_{65}} \tag{67}$$

Now if $\theta_{34} + \theta_5 + \theta_6 = 2n\pi$, and $S_{65} \cong 1$, we find from eq. (67) that $R_r = \infty$ for an value of S_{66} except 0. This was impossible for a reciprocally-coupled ring since in this case $\underline{S}_{14} = \underline{S}_{41}$ must be greater than zero for the steady state amplitude of the ring waves to be greater than zero.

The reflection coefficient seen by the source at the input to the circuit is simply $R_i = b_1/a_1$. This can be calculated in the following manner. From the scattering matrix for the coupler and eq. (64) we can write

$$b_1 = \underline{S}_{14}\, e^{-j\theta_{14}}\, a_4 = \frac{S_{66}\, e^{-j(\theta_{14} + 2\theta_6)}\, b_4}{\left[1 - \sqrt{1 - \underline{S}_{14}^{2}}\; e^{-j(\theta_{34} + \theta_5 + \theta_6)}\, S_{65}\right]} \tag{68}$$

But b_4 is related to a_1 in eq. (65) so that we can write

$$R_i = \frac{b_1}{a_1} = \frac{\underline{S}_{41}\, \underline{S}_{14}\, S_{66}\, e^{-j(\theta_{14} + \theta_{41} + 2\theta_6)}}{E} \tag{69}$$

where E is defined in eq. (62).

Note that we can choose $\underline{S}_{41}^{2} = 1 - A^2 \neq 0$ so that there is a power build-up in the ring, while choosing $\underline{S}_{14} = 0$ so that $R_i = 0$. Thus eq. (69) demonstrates that the source can be isolated from the ring reflections by using a non-reciprocal directional coupler. The properties claimed at the outset for the non-reciprocal directional coupler used in a TWR circuit have been demonstrated in a quantitative fashion and should accurately predict the outcome of experimental tests in most practical cases.

C. Transients in the Absence of Reflections

The analysis of transients in TWR circuits is of considerable theoretical importance, since it is useful in explaining how powers greater than that supplied by the source can be created in TWR circuits. The technique of analysis developed by the author in an earlier paper readily applies to the non-reciprocally-coupled TWR circuit. Althought the discussion given by the author is too lengthy to reproduce here, the approach can be very simply stated. If T^* seconds are required for energy to completely circle the closed path of electrical length D, then the energy arriving at port 3 of the coupler at time t, associated with the wave of amplitude $a_3(t)$, is that energy which traveled around the closed path after leaving port 4 of the coupler at time $t-t^*$, associated with the wave of amplitude $b_4(t-T^*)$. In its trip around the closed path, the wave has been attenuated by a factor A, and phase shifted D electrical degrees due to the length of the path plus ωt^* electrical degrees due to the time T^* required for the trip.

From the preceding description of the transient process, one can derive a set of linear, first order, finite difference equations, with variable coefficients which describe the build-up and decay of the forward and backward waves in the closed path.

Since we have assumed in this section no sources of reflection present in the closed path, the forward and backward waves would be uncoupled. Thus reflections from the dummy load (if present) must supply the energy for the build-up of the backward waves.

If T is the number of seconds after one first began to observe the state of affairs in the closed path, where $T = NT^* + \Delta t$, $0 \leq \Delta t/\_T^*$ (with N equal to the number of times the energy has completely circled the closed path) then one can obtain for the forward waves from the finite difference equation

$$\left(b_4\right)_N = \underline{S}_{41}\, e^{-j\theta_{41}} \left(a_1\right)_N + \sqrt{1-\underline{S}_{41}^2}\; e^{-j(\theta_{34} + \omega T^* + D)} \left(b_4\right)_{N-1} \tag{70}$$

and for the backward waves

$$\left(b_3\right)_N = \underline{S}_{14}\, e^{-j\theta_{14}} \left(a_2\right)_N + \sqrt{1-\underline{S}_{14}^2}\; e^{-j(\theta_{43} + \omega T^* + D)} \left(b_3\right)_{N-1} \tag{71}$$

These difference equations together with the appropriate initial conditions (determined by the particular choice of excitation) can be readily solved for a number of interesting cases. One very simple case is one wherein we have a time-harmonic excitation and no energy stored initially in the circuit. The the forward wave gain as a function of time (or counting number N) is:

$$M_f\left[N\right] = \left|\frac{b_3\left[N\right]}{a_2\left[N\right]}\right| = \left|\frac{\underline{S}_{41}}{1 - A\sqrt{1-\underline{S}_{41}^2}\; e^{jD}}\right| \left|1 - e^{(N+1)\ln\left[A\sqrt{1-\underline{S}_{41}^2}\; e^{jD}\right]}\right| \tag{72}$$

while the backward wave gain M_b is M_f with $\underline{S}_{41}$ replaced by $\underline{S}_{14}$. Notice that

$$\lim_{T\to\infty} M_f = \left|\frac{\underline{S}_{41}}{1 - A\sqrt{1-\underline{S}_{41}^2}\; e^{jD}}\right| = \text{steady state forward gain} = \left(M_f\right)_{s.s.} \tag{73}$$

and

$$\lim_{T\to\infty} M_b = \left|\frac{\underline{S}_{14}}{1 - A \quad 1-\underline{S}_{14}^2\; e^{jD}}\right| = \text{steady state backward gain} = \left(M_b\right)_{s.s.} \tag{74}$$

M(N) is an escalator funtion approaching $M_{s.s.}$ as T goes to infinity (see Fig. 6).

Alternately, if energy is initially in the circuit in the forward and backward waves and no excitation is present, then the decay factors are (see Fig. 7).

$$D_f\left[N\right] \equiv \left| \frac{b_3\left[N\right]}{b_3\left[0\right]} \right| = e^{(N+1)\ln\left[A\sqrt{1-\underline{S}_{41}^{2}}\right]} \tag{75}$$

and D_b equals D_f with $\underline{S}_{41}$ replaced by $\underline{S}_{14}$.

D(N) is an escalator function for which

$$\lim_{T\to\infty} D_f\left[N\right] = \lim_{T\to\infty} D_b\left[N\right] = 0 \tag{76}$$

If we write the decay factors as

$$D_f\left[N\right] = e^{(N+1)\ln\left[A\sqrt{1-\underline{S}_{14}^{2}}\right]} = e^{(N+1)\,\omega T^*/2Q_{Lf}} \tag{77}$$

$$D_b\left[N\right] = e^{(N+1)\ln\left[A\sqrt{1-\underline{S}_{41}^{2}}\right]} = e^{(N+1)\,\omega T^*/2Q_{Lb}} \tag{78}$$

then it is apparent the loaded Q's, Q_{Lf} and Q_{Lb}, are unequal if $\underline{S}_{14} \neq \underline{S}_{41}$, since

$$Q_{Lf} = \frac{-\omega T^*}{2\ln\left[A\sqrt{1-\underline{S}_{41}^{2}}\; e^{jD}\right]} \tag{79}$$

$$Q_{Lb} = \frac{-\omega T^*}{2\ln\left[A\sqrt{1-\underline{S}_{14}^{2}}\; e^{jD}\right]} \tag{80}$$

If L is the length of the closed path and V_g is the velocity of energy transmission around the circuit and is a constant, then $T^* = L/V_g$. If V_p is the phase velocity, X_g is the wavelength in the transmission line of which the closed loop is constructed, $p = L/X_g$, and X the free space wave length, then for the class of transmission lines for which V_p/V_g equals X_g/X, we can write

$$Q_{Lf} = \frac{-p\pi\,(X_g/X)^2}{\ln\left[A\sqrt{1-\underline{S}_{41}^{2}}\; e^{jD}\right]} \tag{81}$$

$$Q_{Lb} = \frac{-p\pi (X_g/X)^2}{\ln\left[A\sqrt{1-\underline{S}_{14}^2}\, e^{jD}\right]} \tag{82}$$

It can be shown that these values of loaded Q are practically identical with the value of loaded Q derived from steady state bandwidth considerations over almost the entire range of the variables.

The unloaded Q's, Q_{ub} and Q_{uf}, are equal. These are obtained by letting $\underline{S}_{14} = \underline{S}_{41} = 0$ and $D = 2\pi n$. Then

$$Q_{uf} = Q_{ub} = \frac{-p\pi (X_g/X)^2}{\ln A} = \frac{p\pi (X_g/X)^2}{gL} \tag{83}$$

where g is the attenuation constant of the transmission lines. Eq. (83) agrees with Golde's result for a reciprocally coupled ring, which he derived from the "energy definition of Q_u".[8]

D: Transients with Reflections Present

One may also analyze, to a limited extent, transients in non-reciprocally-coupled TWR circuits when reflections are present by applying an approach developed by the author in an earlier paper for reciprocally-coupled circuits.[9]

We shall assume:

1. the time spent by any wave within the scatter can be neglected;
2. there is a single source of reflections centrally located in the closed path, in the sense that the times required to traverse θ_5 and θ_6 are equal;
3. the scatterer in the circuit is described by the scattering matrix in eq. (50)

Based on these assumptions one can obtain a set of finite difference equations for the forward and backward wave at port 4.

$$(b_4)_N = \underline{S}_{41}\, e^{j\theta_{41}} (a_1)_N + \sqrt{1-\underline{S}_{41}^2}\,\sqrt{1-\underline{S}_{14}^2}\, S_{55}\, e^{-j(\theta_{34}+\theta_{43}+2\theta_5)} (a_4)_{N-1} + \sqrt{1-\underline{S}_{41}^2}\, S_{56}\, e^{-j(\theta_{43}+\theta_5+\theta_6)} (b_4)_{N-1} \tag{84}$$

and

$$(a_4)_N = S_{65}\sqrt{1-\underline{S}_{14}^2}\, e^{-j(\theta_{34}+\theta_5+\theta_6)} (a_4)_{N-1} + S_{66}\, e^{-2j\theta_6} (b_4)_{N-1} \tag{85}$$

Now if we write

$$V_N = \begin{bmatrix} (a_4)_N \\ (b_4)_N \end{bmatrix} ; \quad G_N = \begin{bmatrix} 0 \\ \underline{S}_{41}\, e^{-j\theta_{41}}\, (a_1)_N \end{bmatrix} \tag{86}$$

and

$$S = \begin{bmatrix} \sqrt{1-\underline{S}_{14}^{\,2}}\; S_{65}\; e^{-j(\theta_{34}+\theta_5+\theta_6+\omega T^*)} & S_{66}\; e^{-j(2\theta_6+\omega T^*)} \\ \sqrt{1-\underline{S}_{14}^{\,2}}\;\sqrt{1-\underline{S}_{41}^{\,2}}\; S_{55}\; e^{-j(\theta_{34}+\theta_{43}+2\theta_5+\omega T^*)} & \sqrt{1-\underline{S}_{41}^{\,2}}\; S_{56}\; e^{-j(\theta_{43}+\theta_5+\theta_6+\omega T^*)} \end{bmatrix} \tag{87}$$

we then have

$$V_N = S \cdot V_{N-1} + G_N \tag{88}$$

Eq. (88) is a linear, first order, finite difference, matrix equation with a variable, matrix coefficient. Its general solution is:

$$V_N = S^{N+1} \cdot V_{\text{initial}} + \sum_{n=0}^{N} S^n\, G_{N-n} \tag{89}$$

Eq. (89) indicates that at some time $T = NT + \Delta t$; the forward and backward waves depend on both the initial forward and backward waves. Further, the forward and backward waves will have buildup and decay components involving the same type of escalator functions encountered earlier.

Conclusions

The general theory of non-reciprocal directional couplers has been developed, and a device which can be used as such a coupler has been discussed.

The use of these couplers in TWR circuits has been extensively explored. The steady state and transient analyses were developed where the effects of reflections were considered. It has been demonstrated that a non-reciprocal variable coupler could be used to protect the source from ring reflections, to isolate the ring from reflections from the dummy load as well as to act as a variable coupler performing all functions at the same time.

In addition, it has been pointed out that the well known theorem, which states that a lossless, matched four port must be a directional coupler, is incorrect.

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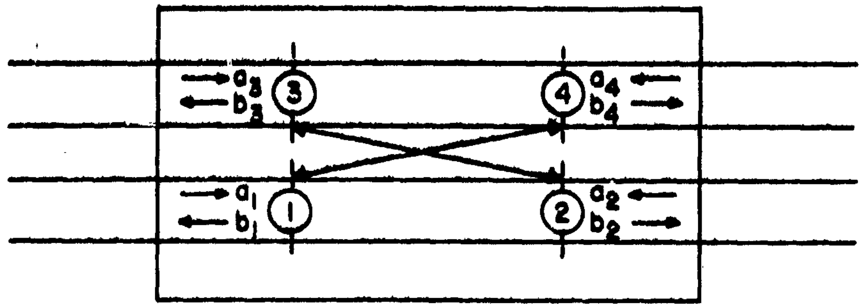

FIG. 1 DIRECTIONAL COUPLER

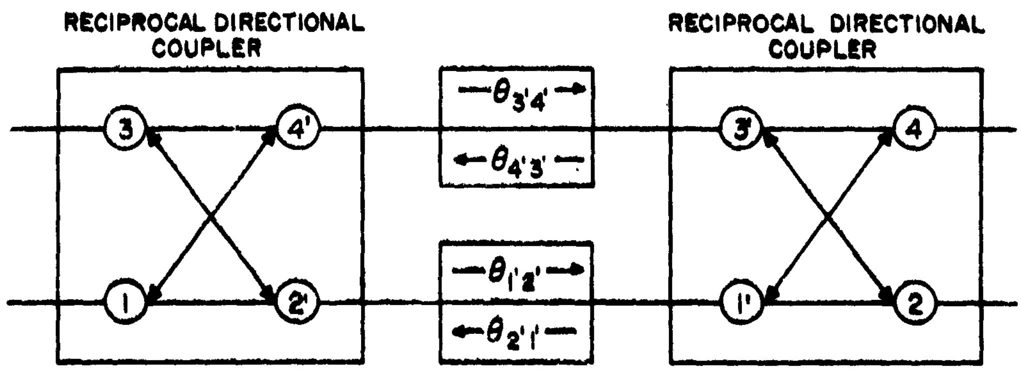

FIG. 2 NON-RECIPROCAL DIRECTIONAL COUPLER

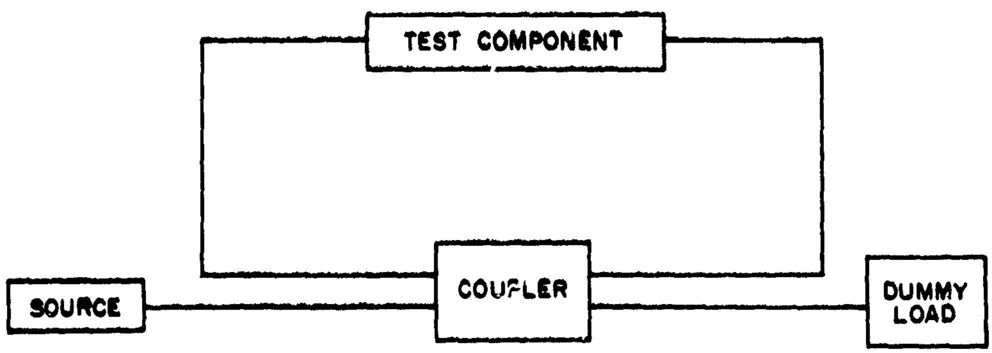

FIG. 3 TRAVELING WAVE RESONATOR WITH AN INTEGRATED TEST COMPONENT

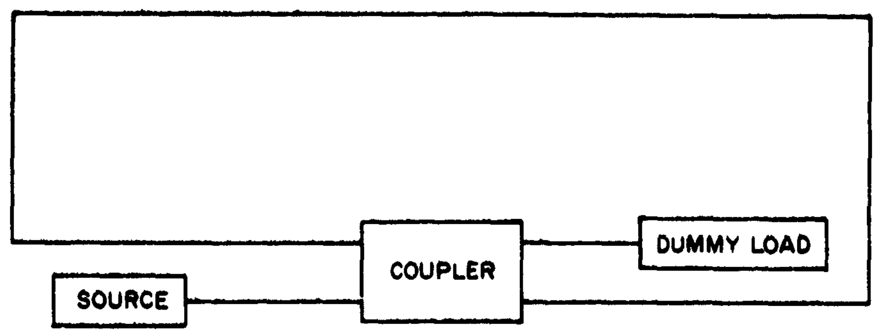

FIG. 4. PRETZEL TWR CIRCUIT

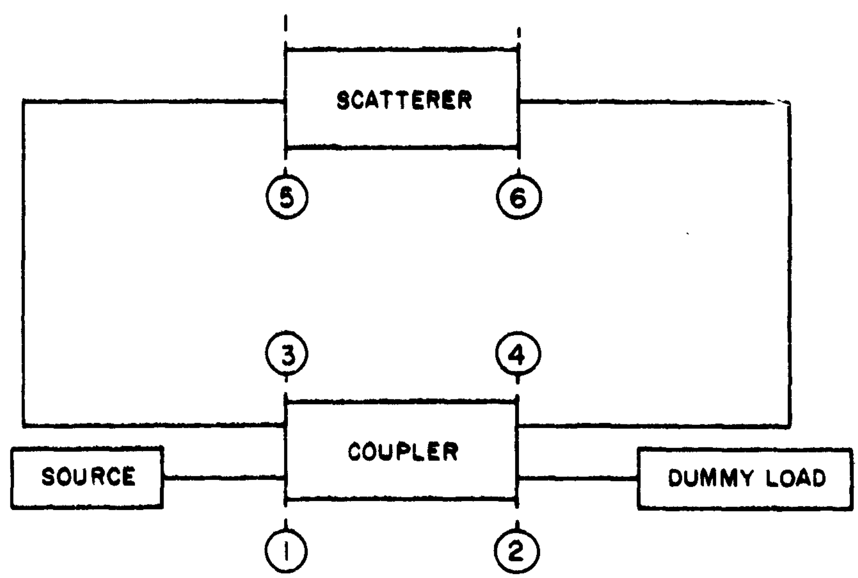

FIG. 5. TWR CIRCUIT WITH SCATTERER

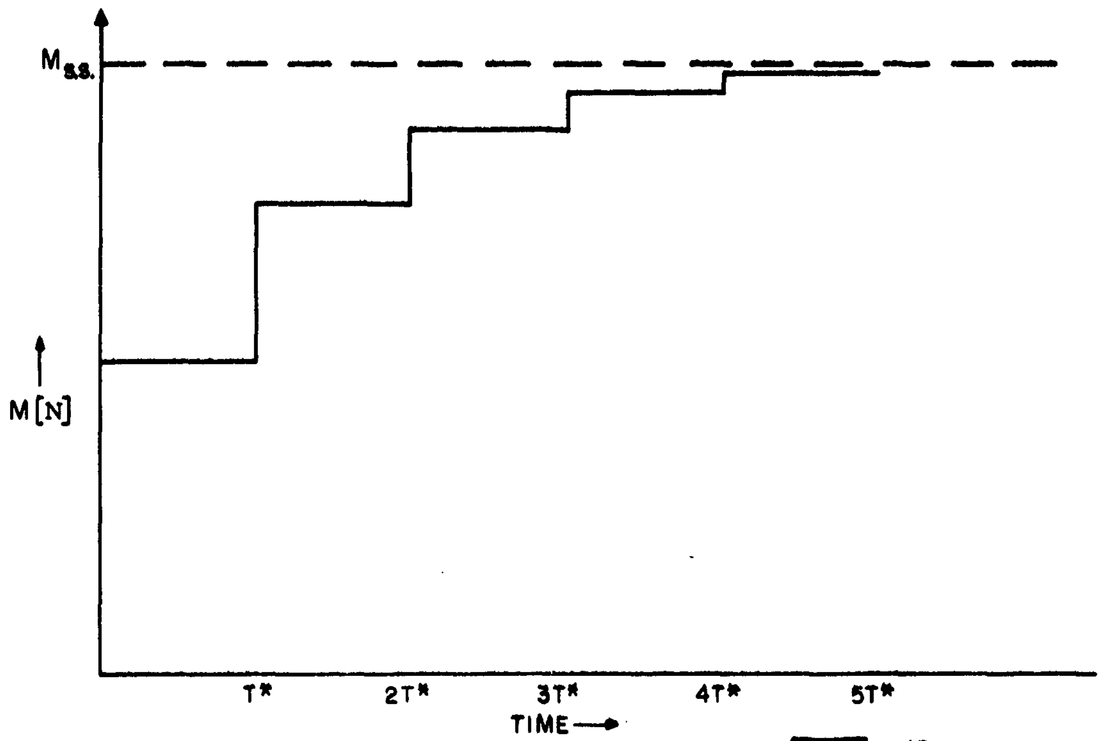

FIG. 6. GAIN OF A TWR CIRCUIT WHEN $A\sqrt{1-\underline{S}_{14}^{2}}\ e^{jD} = 1/2$

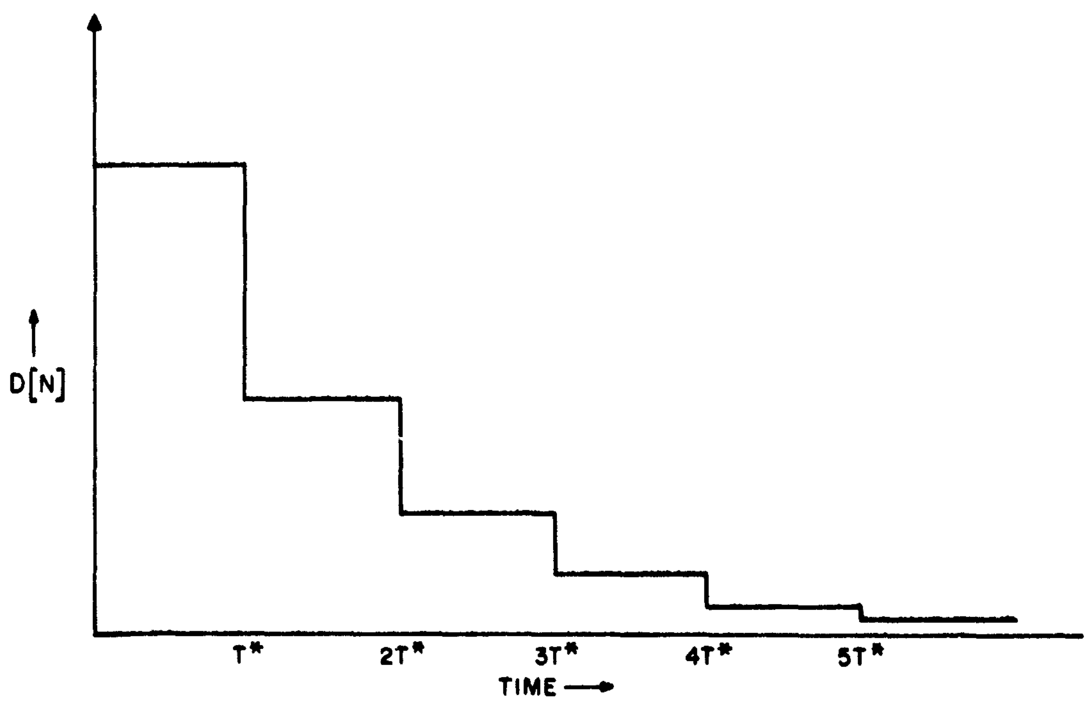

FIG. 7. DECAY IN A TWR CIRCUIT WHEN $A\sqrt{1-\underline{S}_{14}^{2}}\ e^{jD} = 1/2$

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